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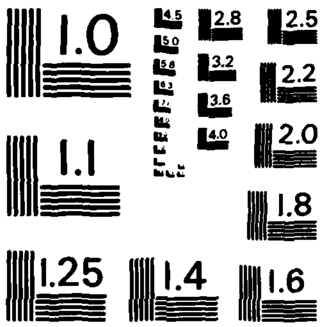

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ON CONSERVATIVE DIFFERENCE SCHEMAS

by

A. A. Samarskiy

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ON CONSERVATIVE DIFFERENCE SCHEMAS

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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ON CONSERVATIVE DIFFERENCE SCHEMAS

A. A. SAMARSKIY

1. The broadening of the scope of difference methods and the associated increase of demands on them stimulate interest in *formulating general* principles of obtaining difference schemas of a definite quality for problems of mathematical physics. The possibility of assigning to the same differential equation an infinite set of difference schemas defined on the same template and having the same order of approximation makes the problem of choosing the desired schema a very complex one. It is required of any numerical method that it supply an approximate solution of the problem with preset accuracy $\varepsilon > 0$ in a finite number of operations. Moreover, it is required that the schema be universal (suitable for a rather broad class of problems), homogeneous, stable, and economical (more precisely, that the numerical algorithm used to solve the difference equations be economical).

All these requirements vie with each other.

Homogeneity of the schema [1] means that the difference operator is defined by the same formula at all grid nodes for any coefficients and right side of the equation of a given functional class, and also for an arbitrary grid.

Below we examine only homogeneous schemas.

2. Approximation error is used as one of the a priori characteristics of a schema. The greatest possible order of approximation of a solution is determined under the assumption of adequate smoothness of the solution. A local (at a point) approximation is *not* generally a very good characteristic of a schema. The negative norm (or "integral norm") (of the form $\|\psi\|_{A^{-1}} = \sqrt{(A^{-1}\psi, \psi)}$, $A = A^*$ positive definite in case of the difference equation $Ay = \psi$), is a natural norm for estimating the error of approximation.

In the case of linear stable schemas, the error of the schema depends continuously on the error or approximation, so that the error of approximation determines the error of the schema. For nonlinear equations (e.g., for gas dynamics equations) such estimates are not available (so far there is no proof of stability of any of the difference schemas for gas dynamics); so here the criterion of approximation plays more or

leas a formal role and may give an improper impression as to the accuracy of a schema owing to strong discontinuities.

Note that comparison of schemas in terms of order of accuracy is meaningful only for sufficiently small steps h and τ: in this connection the higher extent of step is multiplied by the maximum of the derivative of a solution of higher order. But in practice very coarse grids are used where asymptotic estimates may not work. It can happen that a schema of first-order accuracy in an actual grid is more accurate than a schema of second-order accuracy.

Thus, the criterion for choosing schemas should be stated as: the schema should give sufficient accuracy on actual grids for the class of problems under consideration.

From this point of view one should take into account qualitative considerations as well as mathematical ones (if there are any).

3. Conservativeness of the difference schema is one such requirement.

What is conservativeness?

For definiteness, we will deal with problems of mechanics of a continuous medium.

In setting about solving such problems in some region by the method of finite differences, one introduces a grid in the region G and replaces the differential equations by difference equations for the grid functions. As a result one gets a mathematical description of a discrete model of the medium. Obviously, the discrete model should reflect the main features of the continuous medium. The properties of a continuous medium are determined by the integral laws of conservation (of momentum, mass, energy, etc.) for any subregion G′ in G.

The differential equations are corollaries of the integral laws of observation. It is natural to require that the difference schemas express the laws of conservation on the grid. The laws of conservation for the entire grid fegion should be algebraic corollaries of the difference equations. Schemas possessing these properties are called conservative.

15-20 years ago the question of whether the schema should be conservative ("divergent") might have been a matter of discussion, but at present there is general agreement that it should be.

Along with conservative difference schemas one also uses conservative differential-difference schemas.

The general method of integral ratios for obtaining conservative differential-difference schemas was suggested by

A.A. Dorodnitsyn [2] and applied for solving multivariate gas dynamics problems. Later this method was further developed by his students [3].

To get conservative difference schemas one can use the integro-interpolational method or the method of balance [1].

A difference operator on the space of grid functions should preserve the basic properties of the differential operator defined on the space of functions of continuous argument. Such properties in the linear case are self-conjugacy and (sign-) definiteness of the operator. Below, with the example of Laplace-operator schemas in an arbitrary regions, we prove a relationship between self-conjugacy and conservativeness. A nonconservative operator is also nonself-conjugate.

The disadvantages of nonconservative schemas are not removable. Condensation of the grid, which one ought to be able to count on in order to increase accuracy, may, in the case of a nonconservative schema, even increase the error of the schema. Example in [1] confirms this for the problem $(ku')' = 0$, $0 < x < 1$, $u(0) = 1$, $u(1) = 0$.

The nonconservative schema

$$[b_i(y_{i+1} - y_i) - a_i(y_i - y_{i-1})]/h^2 = 0, \quad i = 1, 2, \ldots, N-1,$$
$$y_0 = 1, \quad y_N = 0, \quad h = 1/N.$$

where $a_i = k_i \; -0,\ 25\ (k_{i+1} - k_{i-1})$, $b_i = k_i + 0{,}25\ (k_{i+1} - k_{i+1})$, diverges in case of piecewise constant coefficient $k(x) = k_I$ for $x < \xi$ and $k(x) = K_{II}$ for $x > \xi$, and this divergence is quite peculiar: the solution $y^k\ (x_i)$ of the difference problem has, as $h \to 0$, a limit $\tilde{u}(x)$ not equal to the exact solution $u(x)$ of the differential problem.

The limit function $\tilde{u}(x)$ is a solution of the problem with the additional condition: at the point $x = \xi$ of discontinuity of $k(x)$ there is put a source of power q_o depending on ξ, k_I, k_{II}, equal to zero only for $k_I = k_{II}$ and becoming infinity for certain k_I, k_{II}, ξ. Thus, verification of the accuracy by condensing the grid may, in this case, lead to the improper conclusion that the *nonconservative* schema converges.

Note that any difference schema generates fictitious sources (drains). Indeed, let $\Lambda y + \varphi = 0$ be a difference schema, u the exact solution of the differential equation $Lu + f = 0$. The discrepancy $\Lambda u + \varphi = \psi$ is the error of approximation; it may be viewed as the density of the fictitious sources.

If the schema is conservative, then $\psi(x)$ is an oscillating function, so that always we have

$$(\psi,\ 1) = \sum_{x \in \omega_h} \psi(x) h \to 0 \ \text{as}\ h \to 0.$$

Indeed, for $(ku')' = -f(x)$, $0 < x < 1$, the balance equation has the form

$$ku'\Big|_0^1 + \int_0^1 f(x)\,dx = 0.$$

The corresponding conservative schema $(a(x)y_{\bar{x}})_x = -\varphi(x)$, $x = ih$, possesses the analogous property on the grid:

$$w_N - w_1 + (\varphi,\ 1) = 0, \qquad w_i = a_i y_{\bar{x},i} = a_i \frac{y_i - y_{i-1}}{h}.$$

For $\psi = \Lambda u + \varphi$ we get

$$(\psi,\ 1) = (au_{\bar{x}})_N - (au_{\bar{x}})_1 + (\varphi,\ 1) = O(h^2),$$

if the schema has second order of approximation.

4. The conservativeness of the homogeneous difference schema for the equation

$$\operatorname{div}(k \operatorname{grad} u) = -f(x), \qquad x = (x_1,\ \ldots,\ x_p)$$

is necessary and sufficient for convergence in the class of discontinuous coefficients. For the one-dimensional case this assertion is proved in [1]. In the multivariate case, for an arbitrary region the convergence of the conservative schema with rate $O(\sqrt{h})$ was proved in [4]. The conservative schema will be given below.

Consider the Dirichlet problem for the Poisson equation in the region $G + \Gamma$ on the plane $x = (x_1, x_2)$:

$$\Delta u = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} = -f(x), \quad x \in G, \quad u|_\Gamma = \mu(x). \qquad (1)$$

For this equation the balance equation is satisfied:

$$\int_\Gamma \frac{\partial u}{\partial n} ds + \int_G f(x)\, dx = 0.$$

Let $\bar{\omega}_h = \omega_h + \gamma_h$ be the grid in $G + \Gamma$ described in [5]. It is uniform (with steps h_1 and x_1 and h_2 in x_2) at strictly interior nodes $x \in \overset{\circ}{\omega}$ and nonuniform at boundary nodes $x \in \omega^*$.

First let us consider the well-known five-point schema [5, 6]

$$\Lambda y = y_{\bar{x}_1 x_1} + y_{\bar{x}_2 x_2} = -f(x) \qquad (2)$$

at regular nodes,

$$\Lambda y = \Lambda_1^* y + \Lambda_2^* y = y_{\bar{x}_1 \hat{x}_1} + y_{\bar{x}_2 \hat{x}_2} = -f(x) \qquad (3)$$

at nonregular nodes, where

$$\Lambda_1^* y = y_{\bar{x}_1 \hat{x}_1} = \frac{1}{\hbar_1}\left(\frac{y^{(+1_1)} - y}{h_1^*} - \frac{y - y^{(-1_1)}}{h_1}\right), \quad \hbar_1 = 0{,}5\,(h_1 + h_1^*), \qquad (4)$$

and h^* is the distance of the nonregular node $x \in \omega^{**}$ from the boundary node $x^{(+1_1)} \in \gamma_h$. If $x^{(-1_1)} \in \gamma_h$ is a boundary node and $x^{(+1_1)}$ is an interior node, then

$$\Lambda_1^* y = \frac{1}{\hbar_1}\left(\frac{y^{(+1_1)} - y}{h_1} - \frac{y - y^{(-1_1)}}{h_1^*}\right).$$

We will show that this schema is not conservative. Introducing the notation

$$(y, v)_* = \sum_{x \in \mathring{\omega}} y(x)\, v(x)\, h_1 h_2 + \sum_{x \in \omega^*} y(x)\, v(x)\, \hbar_1 \hbar_2, \qquad (5)$$

we get

$$(\Lambda y,\ 1)_* = \sum_{x \in \omega^*} (y_{n_1} h_2 + y_{n_2} h_1) + \Delta,$$

$$\Delta = \sum_{x \in \omega^*} [\Lambda_1 y \hbar_1 (\hbar_2 - h_2) + \Lambda_2 y \hbar_2 (\hbar_1 - h)],$$

where $y_{n_\alpha} = y_{x_\alpha}$ if $x^{(+1\alpha)}$ is a boundary node, $y_{n_\alpha} = -y_{\bar{x}_\alpha}$ if $x^{(-1\alpha)}$ is a boundary node, $\alpha = 1, 2$.

Comparing (2) and (3), we see that

$$\sum_{x \in \omega^*} (y_{n_1} h_2 + y_{n_2} h_1) + (\varphi,\ 1)_* = -\Delta, \qquad \Delta \neq 0,$$

i.e., the schema (1) is not conservative.

We find similarly that Λ is nonself-conjugate, i.e.,

$$(\Lambda y,\ v)_* \neq (y,\ \Lambda v)_*,$$

where y and v are arbitrary functions defined on $\omega_h + \gamma_h'$ and vanishing on the boundary of γ_h' .

One can find a nonconvex region and a grid such that $-\Lambda$ is not positive definite.

Using the integro-interpolational method [1], one can get the five-point conservative schema [4]:

$$\tilde{\Lambda} y = -f(x), \quad x \in \omega_h, \quad y|_{\gamma_h} = \mu(x). \qquad (6)$$

At regular nodes, $\tilde{\Lambda}$ has the usual form (2), while at nonregular nodes it is given by:

$$\tilde{\Lambda} y = \tilde{\Lambda}_1^* y + \tilde{\Lambda}_2^* y, \qquad (7)$$

where

$$\tilde{\Lambda}_1^* y = \frac{1}{\hbar_1}\left(\frac{y^{(+1_1)} - y}{h_1^*} - \frac{y - y^{(-1_1)}}{h_1}\right) = \frac{1}{\hbar_1}(y_{x_1} - y_{\bar{x}_1}) = y_{\bar{x}_1 \hat{x}_1}, \qquad (8)$$

$$\tilde{\Lambda}_2^* y = \frac{1}{\hbar_2}\left(\frac{y^{(+1_2)} - y}{h_2^*} - \frac{y - y^{(-1_2)}}{h_2}\right) = y_{\bar{x}_2 \hat{x}_2}, \qquad (i)$$

if $x^{(+1_1)}$ and $x^{(+1_2)}$ are boundary nodes. It is not hard to see that

$$\tilde{\Lambda}_1^* = \frac{\hbar_1}{h_1}\Lambda_1^*, \quad \tilde{\Lambda}_2^* = \frac{\hbar_2}{h_2}\Lambda_2^*,$$

where Λ_1^* and Λ_2^* have the form (4).

On the set Ω of grid functions $y(x_1, x_2)$ defined on the grid $\omega_h + \gamma_h$ and equal to zero on the boundary of γ_h, introduce the scalar product

$$(y, v) = \sum_{x \in \omega_h} y(x) v(x) h_1 h_2.$$

The above-defined operator $\tilde{\Lambda}$ is self-conjugate on Ω:

$$(\tilde{\Lambda} y, v) = (y, \tilde{\Lambda} v) \text{ for any } y, v \in \Omega,$$

while $\tilde{\Lambda}$ is positive definite on an arbitrary grid in the case of an arbitrary region.

At nonregular nodes, $\tilde{\Lambda}$ has zero order of approximation:

$$\tilde{\Lambda}u - Lu = O(1) \quad \text{for } x \in \omega^{**}.$$

However, the schema (6) has second-order accuracy

$$\|y - u\|_C = O(|h|^2), \; |h|^2 = h_1^2 + h_2^2.$$

For the equation

$$\operatorname{div}(k \operatorname{grad} u) = -f(x), \; x \in G, \quad u|_\Gamma = \mu(x), \quad k(x) \geqslant c_1 > 0$$

the corresponding conservative schema has, at regular nodes, the form

$$\bar{\Lambda}y = (a_1 y_{\bar{x}_1})_{x_1} + (a_2 y_{\bar{x}_2})_{x_2} = -f(x),$$

at nonregular nodes the form

$$\tilde{\Lambda}y = (a_1 y_{\bar{x}_1})_{\hat{x}_1} + (a_2 y_{\bar{x}_2})_{\hat{x}_2} = -f(x),$$

where

$$(a_\alpha y_{\bar{x}_\alpha})_{\hat{x}_\alpha} = \frac{1}{\hbar_\alpha}\left(a_\alpha^{(+1_\alpha)} \frac{y^{(+1_\alpha)} - y}{h_\alpha^*} - a_\alpha \frac{y - y^{(-1_\alpha)}}{h_\alpha}\right),$$

if $x^{(+1_\alpha)} \in \gamma_\alpha$, $\alpha = 1, 2$. The coefficients a_α are chosen according to [5].

5. Now let us turn to the problem of conservative schemas for equations of magnetic hydrodynamics [7, 8].

We first consider gas dynamics equations which, in Lagrange variables in the plane, have the form:

$$\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x}, \quad \frac{\partial r}{\partial t} = v, \quad \frac{\partial \eta}{\partial t} = \frac{\partial v}{\partial x}, \qquad (10)$$

$$\frac{\partial \varepsilon}{\partial t} = -p\frac{\partial v}{\partial x}, \quad p = p(\eta, \varepsilon), \qquad (11_1)$$

where t is time, r is Euler's coordinate, x is the Lagrange mass coordinate, η is specific volume, p is pressure, ε is intrinsic energy, v is velocity.

The nondivergent equation (11_1) can be written in entropy form

$$\frac{\partial \varepsilon}{\partial t} = -p\frac{\partial \eta}{\partial t}, \qquad (11_2)$$

and, by using equations (10), can also be transformed into the divergent form

$$\frac{\partial}{\partial t}\left(\varepsilon + \frac{v^2}{2}\right) = -\frac{\partial}{\partial x}(pv), \qquad (11_3)$$

expressing the law of conservation of total energy. The three systems of equations (10), (11_α), α = 1, 2, 3, are equivalent.

It is ordinarily assumed (see [9, Ch.III]) that to obtain a conservative schema it is sufficient to approximate the three

basic laws of conservation (balance)--of mass, momentum and total energy. However, in this case the balance equations may be violated for the individual forms of energy--intrinsic, kinetic. The amount of disbalance in the case of strongly varying solutions may become comparable with the total energy.

We will call a difference schema completely conservative if both the laws of conservation of mass, momentum and total energy as well as the individual balance of energy, kinetic and intrinsic, are valid. To obtain such schemas, in addition to requiring conservativeness, a formal condition is imposed: the difference schema should possess the same property as the system of differential equations (10), (11_α), namely the "divergent" difference equation for energy (the analog of (11_3)) should be transformable both to the nondivergent form (to the analog of (11_1)) as well as to the entropy form (11_2).

A five-point family of schemas is examined on the six-point template ($x_i = ih$, $t_j = j\tau$), (x_i, t_{j+1}), ($x_{i\pm1}$; t_j), ($x_{i\pm1}$, t_{j+1}). By requiring the indicated conditions to hold, we get a one-parameter family of completely conservative schemas

$$v_t = -p_x^{(\sigma)}, \quad r_t = v^{(0.5)}, \quad \eta_t = v_x^{(0.5)}, \quad \varepsilon_t = -p^{(\sigma)} v_x^{(0.5)}, \qquad (12)$$

where we have used the notation $f^{(\alpha)} = \alpha \hat{f} + (1-\alpha)f$, $f = f^j$, $\hat{f} = f^{j+1}$, $f_t = (\hat{f}-f)/\tau$, α an arbitrary number. Here $v = v(x_i, t_j)$, $p = p(x_{i-1/2}, t_j)$, $\eta = \eta(x_{i-1/2}, t_j)$, $\varepsilon = \varepsilon(x_{i-1/2}, t_j)$.

The nondivergent energy equation $\varepsilon_t = -p^{(\alpha)} v_x^{(0,5)}$ is transformed to the divergent form

$$\left(\varepsilon + \frac{v^2}{2}\right)_t = -(p^{(\alpha)}(-1)\, v^{(0,5)})_x, \quad p(-1) = p(x-h), \qquad (13)$$

if one uses the motion equation $v_t = -p_{\bar{x}}^{(\alpha)}$ and the formula for difference differentiation of the product $(f(-1)v)_x = f v_x + v f_{\bar{x}}$.

Indeed, we have

$$\varepsilon_t = -p^{(\alpha)} v_x^{(0,5)} = -(p^{(\alpha)}(-1)\, v^{(0,5)})_x + p_{\bar{x}}^{(\alpha)} v^{(0,5)} =$$
$$= -(p^{(\alpha)}(-1)\, v^{(0,5)})_x - v_t v^{(0,5)} = -(p^{(\alpha)}(-1)\, v^{(0,5)})_x - 0.5\,(v^2)_t.$$

Whence (13).

The schema (12) has approximation $O(\tau + h^2)$ for any α. Taking $\alpha = 0{,}5$, we get a unique schema $O(\tau^2 + h^2)$ (it is also derived by somewhat different arguments in [10]).

7. Now we turn to a system of equations of magnetic hydrodynamics. Let $H = H_x(x)$ and $E = E_y(x)$ be the nonzero components of magnetic and electric fields.

In Lagrange variables the system has the form

$$\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x} + F, \quad F = -\sigma\tau_i E H, \quad \frac{\partial r}{\partial t} = v, \quad \frac{\partial \eta}{\partial t} = \frac{\partial v}{\partial x},$$

$$\frac{\partial}{\partial t}(H\eta) = \frac{\partial E}{\partial x}, \quad E = \frac{1}{4\pi\sigma\eta}\frac{\partial H}{\partial x}, \quad \frac{\partial \varepsilon}{\partial t} = -p\frac{\partial v}{\partial x} + Q, \quad Q = \sigma\eta E^2,$$

where F is the Lorentz force, Q is the Joule heat, σ is the electric conductivity.

The energy equation can be replaced by the equation in entropy form

$$\frac{\partial \varepsilon}{\partial t} = -p\frac{\partial \eta}{\partial t} + Q$$

and by the divergent equation for total energy

$$\frac{\partial}{\partial t}\left(\varepsilon + \frac{v^2}{2} + \frac{H^2\eta}{8\pi}\right) = -\frac{\partial}{\partial x}\left[v\left(p + \frac{H^2}{8\pi}\right) - \frac{EH}{4\pi}\right].$$

The force F can be also written in the divergent form

$$F = -\sigma\eta EH = -\frac{\partial}{\partial x}\left(\frac{H^2}{8\pi}\right).$$

Thus, we obtain six equivalent systems of equations.

We will require that the analogous equivalence also hold for the difference schemas. Examining the eight-parameter family of schemas on the above-indicated six-point template, we get the two-parameter family of schemas $O(\tau + h^2)$ and the lone schema $O(\tau^2 + h^2)$ (for $\alpha = \beta = 0{,}5$):

$$v_t = -p_{\bar{x}}^{(\alpha)} - \left(\frac{H\hat{H}}{8\pi}\right)_{\bar{x}}, \quad r_t = v^{(0,5)}, \quad \eta_t = v_x^{(0,5)},$$
$$(H\eta)_t = E_x^{(\beta)}, \quad H_{\bar{x}} = 4\pi\sigma_*\eta_* E,$$
$$\varepsilon_t = -p^{(\alpha)}v_x^{(0,5)} + \frac{1}{2}\left[(\sigma_*\eta_* E)^{(0,5)} E^{(\beta)} + (\sigma(+1)\eta(+1)E(+1))^{(0,5)} E^{(\beta)}(+1)\right].$$

where

$$f_{*} = 0.5\,(f + f(-1)), \qquad f(\pm 1) = f(x \pm h,\ t).$$

The energy equation can be transformed to the divergent form

$$\left(\varepsilon + \frac{1}{4}\,(v^2 + v^2(+1)) + \frac{H^{2\tau}}{8\pi}\right)_t = -\left[\left(p_{*}^{(\sigma)} + \frac{(H\hat{H})_{*}}{8\pi}\right)v^{(0,5)} + \frac{H_{*}^{(0,5)}E^{(\beta)}}{4\pi}\right]_x .$$

It should be noted that conservative schemas approximating the total energy equation may poorly approximate the equations for the intrinsic energy and for magnetic field. This is no less dangerous than violation of the law of conservation of total energy, and it may lead to an erroneous computation of temperature, particularly to such a nonphysical effect as the diminution of the temperature of some mass of gas in the process of compression or in the *presence* of Joule heating. The disbalances arising here cannot be eliminated by spatially condensing the grid.

In the case of implicit schemas, to solve difference equations one uses iterative methods, which may generally violate the conservativeness of the schema. Hence the iterations must be carried through for the prescribed accuracy which characterizes the amount of disbalance. In this case, to simplify the calculations it is advisable to use the energy equation in nondivergent form.

For nonlinear difference schemas there is an amply developed theory which allows general methods to be formulated for obtaining set-quality difference schemas.

Let us point out two rather general methods:

1) The method of regularization of difference schemas based on using the class of stable schemas and the possible of varying, without affecting stability, one of the operators of the schema so as to satisfy the collateral constraints of efficiency and approximation;

2) The method of summary approximation based on using a new notion of a schema and a new notion of approximation for schemas--summary approximation (such schemas are called adaptive).

Both methods are used successfully, particularly for obtaining economical difference schemas in the case of multivariate problems of mathematical physics. A presentation of these methods is given in [11]. Note that in all cases, conservativeness of the schema is an ever-present requirement.

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